

Quantum Entanglement of Neutrino Pairs

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Abstract

It is practically shown that a pair of neutrinos from tau decay can form a flavor entangled state. With this kind of state we show that the locality constraints imposed by Bell inequality are violated by the quantum mechanics, and an experimental test of this effect is feasible within the earth's length scale. Theoretically, the quantum entanglement of neutrino pairs can be employed to the use of long distance cryptography distribution in a protocol similar to the BB84.

Entanglement represents the essence of quantum mechanics (QM) and is the source of a number of paradoxical and counterintuitive phenomena [1]. The kernel of entanglement is the non-local correlation exhibit by QM. In 1964, Bell derived out a group of inequalities [2], that is a set of constraints that local hidden variable theory (LHVT) must satisfy whereas QM violates. Many experiments in regard of the Bell inequalities have been carried out by using entangled photons [3]. Great progress was made in this direction by using PDC technique in generating the entangled photon pairs [4]. More importantly quantum information theory has been developed along with the research of entanglement. The same effort has also been made in high energy physics [5, 6], where the mixing properties of neutral mesons can play a practical role [7, 8, 9]. People also attempted to use the entangled state in high energy physics to implement quantum information tasks [10], i.e. quantum teleportation.

Neutrinos are one class of elementary particles known as having only weak interaction with matter. This nature enables them to transmit over a large scale of distance without significant attenuation. For this reason, thirty years ago it was proposed to be as an ideal

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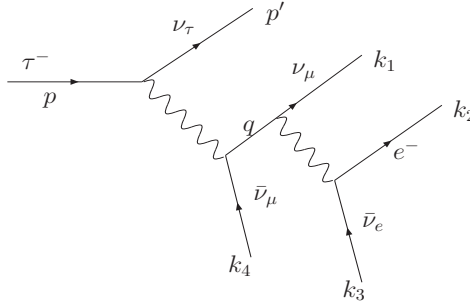


Figure 1: The tau lepton decay process, $\tau^- \rightarrow \nu_\tau + \nu_\mu + e^- + \bar{\nu}_\mu + \bar{\nu}_e$, at leading order.

means to transmit signals [11]. In the well-established elementary particle theory, the Standard Model, there are three kinds(flavors) of neutrinos. The neutrinos with different flavors can oscillate from one to another during the period of transmission. A nature question then is: can neutrinos form an entanglement state and exhibit the nonlocal correlations? We notice that in literatures there are some discussions on the possible EPR effects in neutrino sector [12, 13]. However, only the momentum variable, but not the flavors, were concerned. We show in this work that two flavors of neutrinos from the tau decay can form an entangled state, which can be used as a test of LHVTs. In addition, we argue that this entangled state can also be employed in quantum key distributions and a simple quantum key distribution protocol is proposed for it.

We can consider the tau lepton decay process $\tau^- \rightarrow \nu_\tau + \nu_\mu + e^- + \bar{\nu}_\mu + \bar{\nu}_e$ for our aim. The dominant contribution to this process comes from the tree level diagram, as shown in Figure 1. In case the two final anti-neutrinos, k_3, k_4 symbolizing their momenta in the figure, emitted in (nearly) parallel, they will have the same helicity(here their masses are neglected), which make their total spin wave function symmetric. To keep antisymmetric nature of fermions, their flavor wave function must be antisymmetric. Thus, the wave function of this quasi-paralleled neutrino pair can be formulated as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{\bar{\nu}_e\bar{\nu}_\mu - \bar{\nu}_\mu\bar{\nu}_e\}. \quad (1)$$

The corresponding differential decay width can be calculated straightforwardly. It reads

$$d\Gamma \approx \frac{6G_F^2}{\pi^5 m_\mu^4} (m_\mu - E/2)(m_\tau - 2E)^2 E^4 dE d\epsilon d\Omega_4 d\Omega_3, \quad (2)$$

where E is the mean energy of these two entangled antineutrinos and ϵ is half of the energy difference of them.

The current global analysis of the neutrino data exhibit that there are only three mixed active neutrinos [14]. According to the tri-bimaximal mixing scheme we have the following

mixing matrix [15, 16]:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \quad (3)$$

The initial entanglement state (1) can therefore be expressed in the mass eigenstate,

$$|\Psi(0)\rangle = \frac{1}{2}\{\nu_1\nu_2 - \nu_2\nu_1\} + \frac{1}{\sqrt{6}}\{\nu_1\nu_3 - \nu_3\nu_1\} + \frac{1}{\sqrt{12}}\{\nu_2\nu_3 - \nu_3\nu_2\}. \quad (4)$$

With the time evolution, the initial state evolves to

$$\begin{aligned} |\Psi(t_l, t_r)\rangle &= \frac{1}{2}\{e^{-i(E_1t_l+E_2t_r)}\nu_1\nu_2 - e^{-i(E_2t_l+E_1t_r)}\nu_2\nu_1\} + \\ &\quad \frac{1}{\sqrt{6}}\{e^{-i(E_1t_l+E_3t_r)}\nu_1\nu_3 - e^{-i(E_3t_l+E_1t_r)}\nu_3\nu_1\} + \\ &\quad \frac{1}{\sqrt{12}}\{e^{-i(E_2t_l+E_3t_r)}\nu_2\nu_3 - e^{-i(E_3t_l+E_2t_r)}\nu_3\nu_2\}, \end{aligned} \quad (5)$$

where t_l and t_r represent the evolving time on left and right sides (one can anyway name two neutrinos to be left and right). In Quantum Mechanics, the coincident-count probability of ν_i being detected at t_l and ν_j being detected at t_r is

$$P(t_l = \nu_i, t_r = \nu_j) = |\langle \nu_i \nu_j | \Psi(t_l, t_r) \rangle|^2, \quad (6)$$

where i, j represent e, μ, τ . Following we show that this QM result cannot be reproduced by a LHVT, i.e., the Bell inequality will be violated.

The Clauser and Horne [17] (CH inequality) type Bell inequality for neutrinos takes the following form

$$\begin{aligned} P(t_{l_2} = \nu_e, t_{r_2} = \nu_\mu) - P(t_{l_2} = \nu_e, t_{r_1} = \nu_e) + P(t_{l_1} = \nu_\mu, t_{r_2} = \nu_\mu) + P(t_{l_1} = \nu_\mu, t_{r_1} = \nu_e) \\ - P(\infty, t_{r_2} = \nu_\mu) - P(t_{l_1} = \nu_\mu, \infty) \leq 0, \end{aligned} \quad (7)$$

where $P(t, \infty)$ means that all three-flavor triggers on the right side are countered. In this case the neutrino flavors play the role of polarizations, and the time variation play the role of angles between them in photon experiment [5]. Inequality (7) is in agreement with a recent work of Ref. [18], in which the Hardy state is generalized to three or more dimensions in the Hilbert space. From [19] we can get the following inequality for the entangled neutrino state, i.e.

$$\begin{aligned} H = \frac{P(t_{l_1} = \nu_\mu, t_{r_2} = \nu_\mu)}{P(\infty, t_{r_2} = \nu_\mu) - P(t_{l_2} = \nu_e, t_{r_2} = \nu_\mu) + P(t_{l_1} = \nu_\mu, \infty) - P(t_{l_1} = \nu_\mu, t_{r_1} = \nu_e)} \\ + \frac{P(t_{l_2} = \nu_e, t_{r_1} = \nu_e)}{P(\infty, t_{r_2} = \nu_\mu) - P(t_{l_2} = \nu_e, t_{r_2} = \nu_\mu) + P(t_{l_1} = \nu_\mu, \infty) - P(t_{l_1} = \nu_\mu, t_{r_1} = \nu_e)} \leq 1. \end{aligned} \quad (8)$$

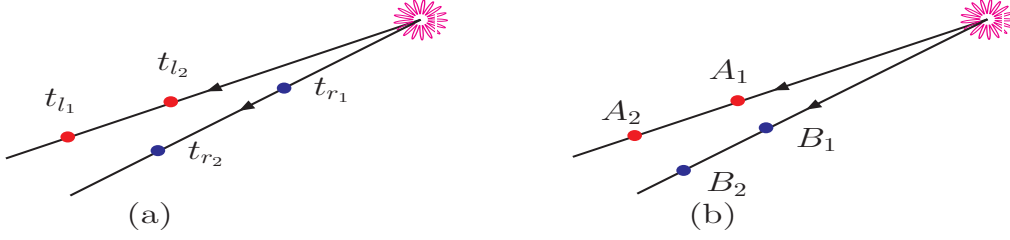


Figure 2: (a) exhibits the configuration for testing the Bell inequality; (b) exhibits the configuration for the use of quantum key distribution.

To evaluate the quantum prediction for the inequality (8), we need the neutrino mixing parameters. In the calculation of Eq.(6), the approximation $\Delta E_{ij} = E_i - E_j = \frac{\Delta m_{ij}^2}{2E}$ can be taken, where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ and $\frac{\Delta m_{ij}^2 L}{2E} \simeq 2.54 \times \Delta m_{ij}^2 \times \frac{L(\text{km})}{E(\text{GeV})}$ with L to be the distance in unit of kilometer. Since we have three neutrino mass eigenstates, there are two independent mass difference squares, i.e. $\Delta m_{21}^2 + \Delta m_{32}^2 + \Delta m_{13}^2 = 0$. We adopt the recent values given by the Particle Data Group(PDG) for further numerical calculation [20]:

$$\Delta m_{21}^2 = 8 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{32}^2 = 2.4 \times 10^{-3} \text{ eV}^2. \quad (9)$$

In nature unit, the relation between neutrino travelling distance L and it energy reads [20]

$$L = \frac{s \times E(\text{GeV})}{2.54} \times 10^5 (\text{km}). \quad (10)$$

Here, s defined as $\frac{L}{2E}$, E is the energy of the neutrinos. Thus in the numerical result s play the role of $t_{l,r}$. We may take $E = 107 \text{ MeV}$ and $s < 0.6$ in the evaluation in order to make the experiment viable at 10^3 km scale.

When taking

$$t_{l_1} = 0.579497, \quad t_{l_2} = 0.0579214, \quad t_{r_1} = 0.0001, \quad t_{r_2} = 0.180264, \quad (11)$$

we get the maximal value of H in (8) to be ~ 1.71 , which apparently violates the inequality. With $t_{l,r}$ in (11), we can get the corresponding detection distances

$$L_1 = 2418 \text{ km}, \quad L_2 = 241.72 \text{ km}, \quad R_1 = 0.42 \text{ km}, \quad R_2 = 752.28 \text{ km}. \quad (12)$$

Figure (3) exhibits a global view of the violations, where two parameter contours are obtained when the other two parameters are fixed at their maximum values.

In practical experiment we need to choose four spacial detection points, or in other words the detection time $t_{l_1}, t_{l_2}, t_{r_1}, t_{r_2}$ with two on each side (see (a) in Figure 2), and measure the probability of coincident counts $P(\nu_i, \nu_j) = \frac{N_{\nu_i, \nu_j}}{N_{total}}$. We assume that detectors have the same detect efficiency η , and notice that the η is a small number for neutrinos. In this

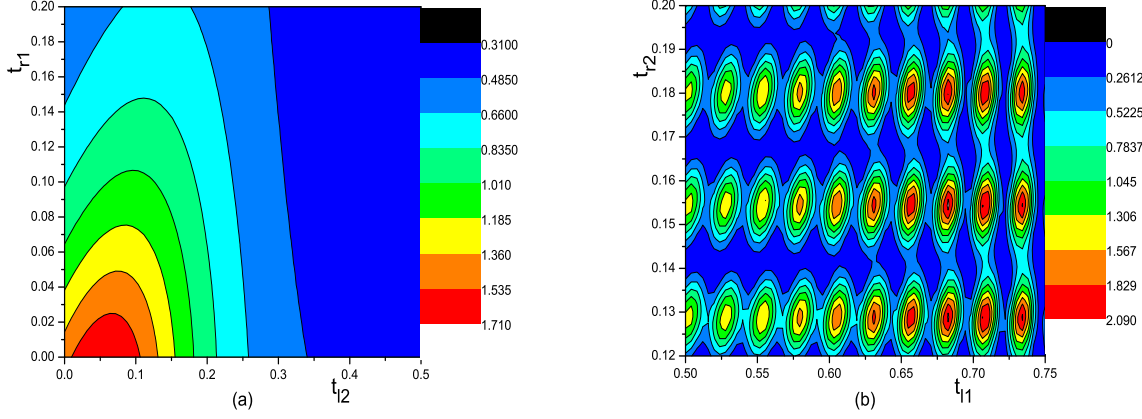


Figure 3: A global view of H in different distance from the source. (a) represents the near-site detectors L_2, R_1 . (b) represents the far-site detectors L_1, R_2 . The red region indicates a violation of QM predictions, where $H > 1$.

condition, the detect efficiency and the total counting number N_{total} can be factorized out of the inequality (8).

From (10) we can get the energy spectral ΔE to be

$$\Delta s = \frac{\Delta E}{E(E + \Delta E)} L \times 2.54 \times 10^{-5} = \frac{\Delta E}{E} s. \quad (13)$$

And, from this we may estimate to what extent the energy spectral affects the inequality violation. Eq.(10) indicates that the low energy corresponds to the short distance of flavor oscillation. Hence, in the tau decay experiment one may choose $E \approx 107\text{MeV}$ for detection, which is just above the muon production threshold.

Notice that in the detection of inequality (8) in experiment, one needs to measure the quantity $P(\infty, t_{r2} = \nu_\mu)$ and $P(t_{l1} = \nu_e, \infty)$. It is obvious that $P(\infty, t_{r2} = \nu_\mu)$ and $P(t_{l1} = \nu_e, \infty)$ are independent of the time on the ∞ side. Therefore, one can choose to measure the quantity $P(\infty, t_{r2} = \nu_\mu)$ and $P(t_{l1} = \nu_e, \infty)$ at some special points, where give the probabilities $P(t_x = \nu_\tau, t_{r2} = \nu_\mu)$ and $P(t_{l1} = \nu_e, t_y = \nu_\tau)$ sufficiently small. For instance, we find that at $t_y = 0.02604$, $P(t_{l1} = \nu_e, t_y = \nu_\tau)$ is of order 10^{-4} ; at $t_x = 0.02568$, $P(t_x = \nu_\tau, t_{r2} = \nu_\mu)$ is of order 10^{-3} . They are both negligible in comparison with the other numbers in the inequality, i.e., $H = \frac{0.318213}{0.18616} = 1.71 \not\leq 1$. This means that one needs only to detect two flavors of neutrinos, the ν_e and ν_μ , by virtue of choosing specially points to place the detectors.

Finally, we show that there exists a natural quantum key distribution protocol for the entangled neutrinos. In the situation of entangled neutrino pairs from the tau lepton decays,

the four detectors can be placed as follows: A_1 to B_1 and A_2 to B_2 have the same distances from the source of neutrino production, as shown in (b) of Figure 2, where A for Alice and B for Bob. The entanglement state makes the coincident counting in A and B to be either $AB = \{\nu_\mu, \nu_e\}$, or $AB = \{\nu_e, \nu_\mu\}$. Whereas the ν_τ will not be counted since the neutrinos are originated from the τ decays. Obviously, the same flavor counts must be precisely zero. Suppose Eve(E) make a measurement on the entanglement state before A and B and resend it. The quantum state now becomes a product state of $\Psi = \bar{\nu}_e \bar{\nu}_\mu$ or $\Psi = \bar{\nu}_\mu \bar{\nu}_e$. In order not to be found by Alice and Bob, E must select certain detection point, where the same flavor counting of $A_1 B_1$ and $A_2 B_2$ are both zero. However, by using of the previous neutrino mixing parameters we find that the zero counting of the same flavors for the product state is a periodical function. Thus, as long as the distance between A_1 and A_2 (so as to B_1 and B_2) is less than that period, Eve can not obtain any information without being found. This protocol is very similar to that of BB84.

In conclusion, we have shown through a practical experimental design, the tau lepton decay, that the entangled (anti)neutrino state can be constructed. We demonstrate that there exists large Bell inequality violation possibility in the detection of the entangled neutrino states, which may be treat as a crucial test of LHVts in high energy physics. Furthermore, we show that there at least exists a protocol, similar to BB84 scheme, for the quantum cryptography distribution. More importantly, the special nature of neutrino makes long-distance quantum communication possible in addition to the effort in electromagnetic approaches. Likely, we will come to the era of neutrino quantum communication in near future.

Acknowledgments

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